The Magnitude and Constancy of Second-Order Extinction at a Low-Altitude Observatory Site

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Abstract

Second-order extinction is routinely assumed to be small, nearly constant, and only significant for B band or (B-V) color index measurements. This study addressed the question, "are these assumptions valid at a typical low-altitude amateur observatory site?" Happily, they are. This result should give comfort to other photometrists who use sites that are not located on high mountain tops.

1. Purpose

For many projects, determination of atmospheric extinction is necessary in order to report the standard (exoatmospheric) photometry of a target object. First-order extinction must normally be determined each night; but there are efficient methods for doing this with a small investment of the night's observing time. The determination of second-order extinction requires a special set of observations, which (if done every night) would use up a significant amount of observing time. Therefore, many photometrists rely on the conventional wisdom that "second-order extinction is small (typically $k''_{bv} \approx 0.04$), it does not vary from night to night, and it is negligible for v and r band." This study of second order extinction had two purposes:

First, the conventional wisdom is validated by experience from professional observatories situated at high altitude sites. But: is second-order extinction small, and is it invariant, when observing at a typical amateur observatory site? Altimira Observatory conducts photometric studies from a location in southern California, near the coastal plain, at only 183 meters ASL. Hence, it seemed worthwhile to study the magnitude and night-to-night variability of second-order extinction.

This aspect of the study was encouraged by Landolt's (ref 1) observation that even under the nearly-ideal conditions of the Cerro Tololo Inter-American Observatory, there is a measurable range of variation in second-order extinction coefficients.

Second, the standard definitions of second-order extinction are based on observations in B and V bands. But with modern CCD's, it is often more convenient to observe in V and R bands. Therefore, an extension of the standard second-order extinction equations to consider observations made primarily in V and R bands has been developed and investigated.

2. Relevant Equations

Following Henden and Kaitchuck (ref 2), and using their nomenclature, the observed v-magnitude of a target is:

$$v = \underbrace{v_0}_{\text{exoatmospheric magnitude}} + \underbrace{k'_v \cdot X}_{\text{1st order extinction: stars dim as they approach the horizon}} + \underbrace{k''_v \cdot (b-v) \cdot X}_{\text{2nd order extinction: blue stars fade faster than red stars, as they approach the horizon}} \quad (1)$$

and the observed b-magnitude is:

$$b = b_0 + k'_b \cdot X + k''_b \cdot (b-v) \cdot X \quad (2)$$

Subtracting Eq. 2 from Eq. 1 gives the (b-v) color index

$$(b-v) = (b-v)_0 + k'_{bv} \cdot X + k''_{bv} \cdot (b-v) \cdot X \quad (3)$$

and shows that the first- and second-order extinction coefficients for color index are related to the extinction coefficients in the individual bands by

$$k'_{bv} = k'_b - k'_v$$

and

$$k''_{bv} = k''_b - k''_v \qquad (4)$$

Although not often seen in photometric reports (because it is presumed to be negligible), the second-order extinction effect on r-magnitude and (v-r) color index are:

$$r = r_0 + k'_r \cdot X + k''_r \cdot (b\text{-}v) \cdot X \qquad (5)$$

and

$$(v\text{-}r)=(v\text{-}r)_0 + k'_{vr} \cdot X + k''_{vr} \cdot (b\text{-}v) \cdot X \qquad (6)$$

where

$$k''_{vr} = k''_v - k''_r$$

In the above equations,
b, v, r = measured instrumental magnitudes
b_0, v_0, r_0 = exoatmospheric instrumental magnitudes
X = air mass of the observation
k′ = first-order extinction coefficients
k″ = second-order extinction coefficients

It is important to note that these second-order extinctions are defined relative to the (b-v) color index of the target, so that the second order extinction is measured in “magnitudes per air mass per magnitude of (b-v) color”. In Section 7, I will extend this definition to consider a second-order extinction that is anchored in the (v-r) color index.

3. Determination of Extinction

3.1 Second-order v-magnitude extinction: The standard method of determining second-order extinction is to measure a red-blue pair of stars at a wide range of air masses. Applying Eq. 1, we expect star #1 to follow the equation:

$$v_1 = v_{0,1} + k'_v \cdot X + k''_v \cdot (b\text{-}v)_1 \cdot X$$

and similarly for star #2:

$$v_2 = v_{0,2} + k'_v \cdot X + k''_v \cdot (b\text{-}v)_2 \cdot X$$

Subtracting these two equations and rearranging gives

$$\Delta(v) = k''_v \cdot \Delta(b\text{-}v) \cdot X + \Delta(v)_0 \qquad (7)$$

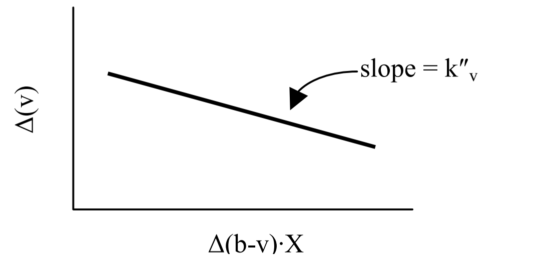
where

$$\Delta(v) = v_1 - v_2$$

and

$$\Delta(v)_0 = v_{0,1} - v_{0,2}$$

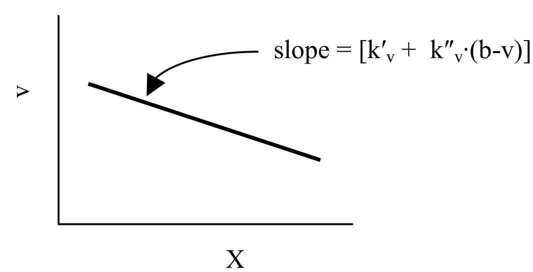
Eq. 7 is linear: a plot of Δ(v) vs. Δ(b-v)·X will be a straight line, whose slope is the second-order extinction, k''_v.



Determination of first-order extinction is then done by applying Eq. 1. Rearranging and grouping the terms gives:

$$v = [k'_v + k''_v \cdot (b\text{-}v)] \cdot X + v_0 \qquad (8)$$

Thus, a plot of v versus X for any single star will be a straight line, whose slope is given by the term in square brackets:



Note that the first order extinction is *not* simply the slope of the plot of v vs. X. Eq. 8 explains something that most photometrists have probably observed in their data. If you use more than one “comp star”, and create plots of v vs. X for each comp star, the lines will have slightly different slopes. This is the signature of second-order extinction on comp stars of different colors.

3.2 Second-order extinction of b- and r-magnitude: For completeness, I note that the same method of section 3.1 can be applied to determine second-order extinction in b and r. The relevant equations (based on following a red-blue pair through a wide range of air mass) are:

$$\Delta(b) = k''_b \cdot \Delta(b\text{-}v) \cdot X + \Delta(b)_0 \qquad (9)$$

$$\Delta(r) = k''_r \cdot \Delta(b\text{-}v) \cdot X + \Delta(r)_0 \qquad (10)$$

The only tricky thing to notice in Eqs. 9 and 10 is that they both use (b-v) as the color index that describes "how blue or red is the star of interest?"

Similar to Eq. 8, the plots of instrumental magnitude vs. air mass (used to determine first-order extinction) in these two bands are described by:

$$r = [k'_r + k''_r \cdot (b\text{-}v)] \cdot X + r_0$$

$$b = [k'_b + k''_b \cdot (b\text{-}v)] \cdot X + b_0$$

so again, the slope of the line of IM vs. X is not the first-order extinction – the term involving k'' and color index must be recognized.

3.3 Second-order extinction of (b-v) color index: Following the same procedure as above, the second-order extinction coefficient for (b-v) color index is found by following a red-blue pair of stars through a range of air mass, and applying Eq. 2:

$$\Delta(b\text{-}v) = k''_{bv} \cdot \Delta(b\text{-}v) \cdot X + \Delta(b\text{-}v)_0 \qquad (11)$$

where

$$\Delta(b\text{-}v) = [(b\text{-}v)_1 - (b\text{-}v)_2]$$

and

$$\Delta(b\text{-}v)_0 = [(b\text{-}v)_{0,1} - (b\text{-}v)_{0,2}]$$

According to Eq. 11, a plot of Δ(b-v) versus Δ(b-v)·X will have a slope equal to the second order extinction, k''_{bv}.

First-order extinction is determined by applying Eq. 2. Rearranging and grouping the terms gives:

$$(b\text{-}v) = [k'_{bv} + k''_{bv} \cdot (b\text{-}v)] \cdot X + (b\text{-}v)_0 \qquad (12)$$

Thus, a plot of (b-v) versus X will be a straight line, whose slope is given by the term in square brackets.

As a practical matter, I find it to be more convenient to determine the second-order extinction separately for b and v (using Eq. 7 and Eq. 9), and then calculate k''_{bv} using Eq. 4.

4. Observed values of first- and second-order extinction

I devoted several nights to monitoring a red-blue star pair at Altimira Observatory. This data set provides some insight into both the typical values of extinction at a low-altitude observatory site, and the constancy of the extinction values.

4.1 Details of 01-18-2005 UT observations and analysis: A typical set of results, which I will present in some detail in order to describe the data analysis methods, is from UT 01-18-2005. The imaging sequence was RR-VV-BB- ··· and since there are a few minutes between exposures, the air mass changes slightly between filters. While it would be possible to construct an "average" color index and "average" air mass bridging adjacent color exposures, it is more convenient to analyze the data in terms of individual colors, and then derive the color-index extinctions.

Figure 1 shows the observed magnitude vs. air mass in b, v, and r bands for six stars. They illustrate the effects predicted by Eqs. 1, and 8:

- the curves quite accurately fit linear trend lines (typical correlation coefficients are $R^2 \approx 0.9$ or larger)
- each star has a unique slope (e.g. looking at the b-band curves, star #3 has a slope = 0.2447, while star #t has a slope = 0.3028).

The raw instrumental color indices of these 6 stars, as measured at air mass ≈ 1.2, are shown in the table below. Star "t" is HD 50279, and star "1" is HD 50167. This pair is one of the recommended "red-blue" pairs for determining second-order extinction. Note that stars 2 and 3 also provide a wide range of color, and so provide a second pair for second-order extinction determination, in the same CCD field of view.

star#	t	1	2	3	4	5
(b-v)	0.63	1.78	0.59	1.85	1.55	0.91

The star pairs (t, 1) and (2, 3) were used for second-order extinction determination in b, v, r bands. The relevant data is plotted in Figure 2. The resulting determinations of second-order and first-order extinction are described in the next section.

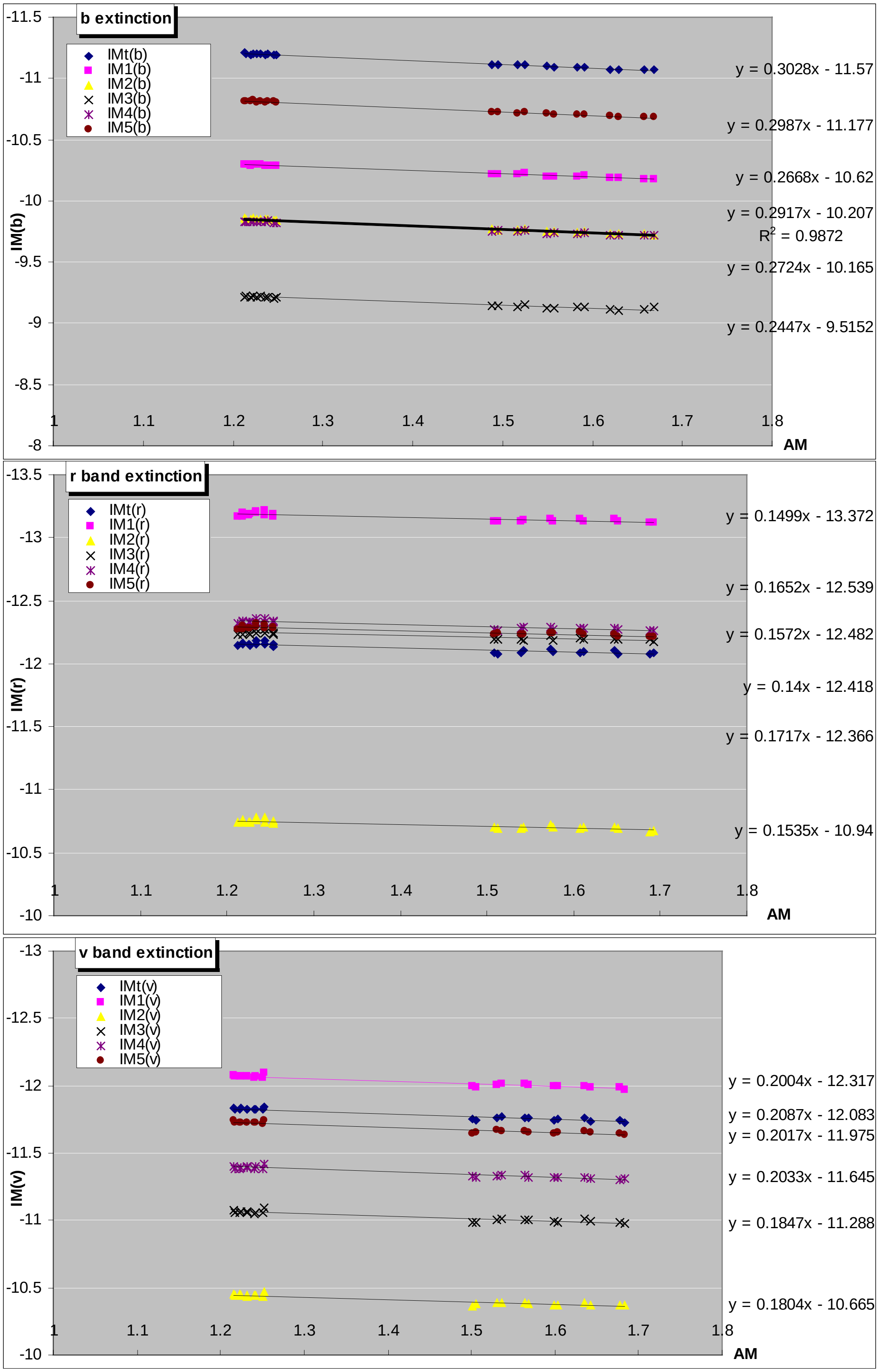


Figure 1: b, r, and v 1st order extinction plots for UT 01-18-2005

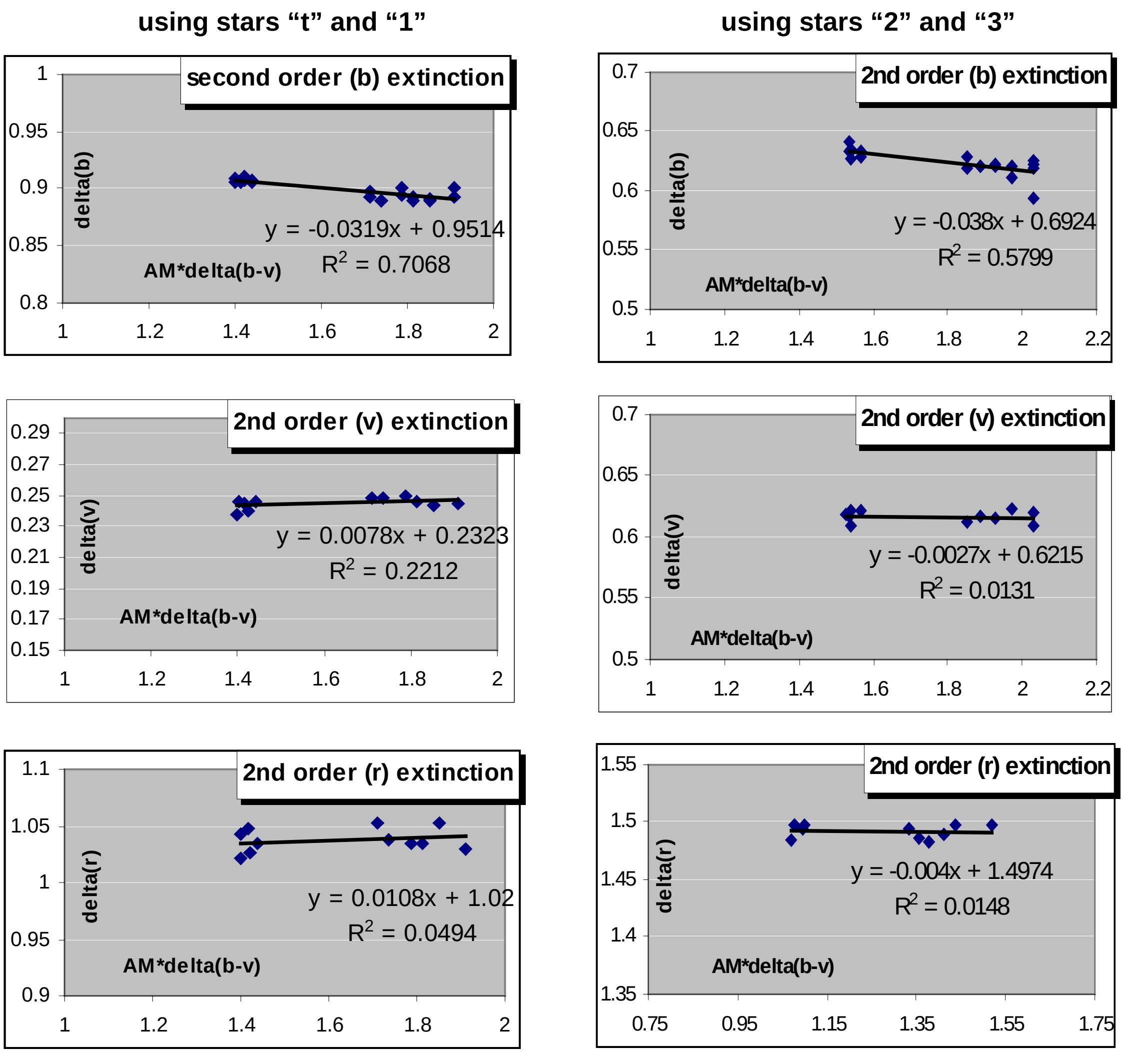


Figure 2: Second-order extinction curves, using star-pair “t” and “1” (left column),

4.2 Extinction Results on 01-18-2005 UT: The data from Figure 2 is used (with Eq. 4) to calculate the second-order extinctions. The results are shown in Table1:

Table 1: Second-order extinctions (from Figure 2 data)

	k''_b	k''_v	k''_r	k''_{bv}	k''_{vr}
using stars (t,1)	-0.03	0.01	0.01	-0.04	0.00
using stars (2,3)	-0.04	0.00	0.00	-0.04	0.00
average =	-0.03	0.00	0.00	-0.04	0.00

Note that the magnitude of second-order extinction for b-band is much larger than any of the others, and that the v- and r-band second-order extinctions are negligible (consistent with the conventional wisdom). Also, it doesn’t matter much whether star pair (t,1) or star pair (2,3) is used – the measured values for k''_b (and k''_{bv}) are similar, and the other values are essentially equal to zero.

In the following section, the results from other nights (calculated in the same way as described here) are combined in a statistical analysis of the night-to-night variation of the second-order extinction coefficients.

5. Night-to-Night Constancy of Second-order Extinction Coefficients

Determination of k′ and k″ on several nights was done to address a few fundamental questions:

- what are "typical" values of second-order extinction for my (low-altitude) observatory site?
- is second-order extinction (approximately) constant from night-to-night?
- does second-order extinction correlate with first-order extinction?

Second-order extinction was determined on 7 nights that appeared subjectively to be "good" nights for photometry – clear, stable, and acceptable seeing.

Figure 3 shows the range of second-order extinction results that were observed.

The second-order extinctions in v and r, and (v-r) are close enough to zero to be negligible for most purposes, in comparison to other error sources. Second-order extinction in b and (b-v) is significant, and shows a modest level of variation from night to night – large enough that use of an "average" value will generate a couple hundredths of a magnitude uncertainty in reported photometry – but using an "average" value is better than ignoring the effect altogether!

There is a hint of a weak correlation between first- and second-order extinction in b-band (i.e. the second-order extinction k''_b is more negative on nights when first-order extinction k'_b is larger), as shown in Figure 4.

This possible correlation needs more data, but if confirmed may offer a way to improve photometric accuracy when using an "average" (i.e. average trend line) value of k''_b.

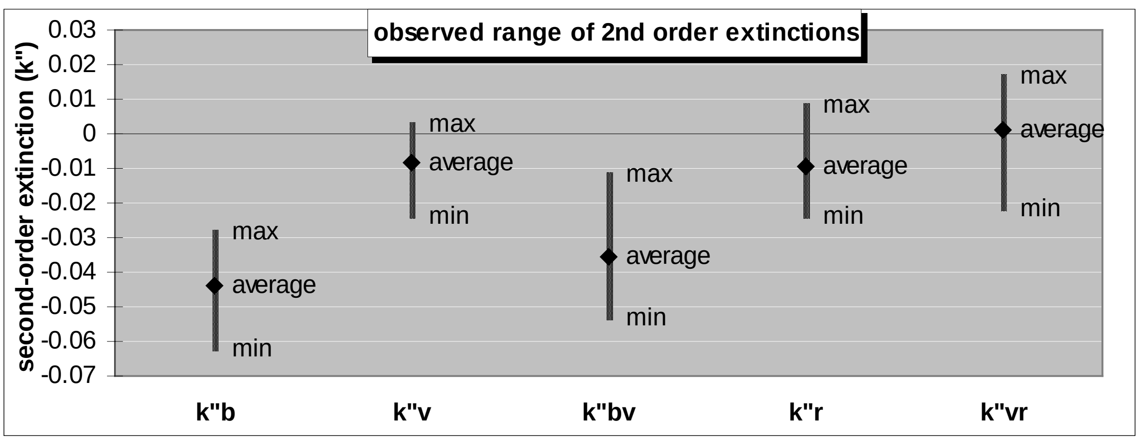


Figure 3: Observed range of Second-Order Extinction

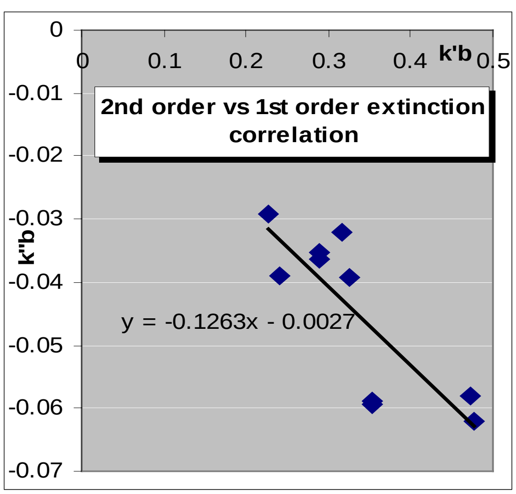


Figure 4: Possible correlation between 1st and 2nd order extinction

6. A way of checking 2nd order extinction using comp star data in the target field

Asteroid light-curve projects require monitoring the target continuously for the entire night. Determination of total extinction $[k' + k''\cdot(b\text{-}v)]$ is thus a "free" benefit – the comp stars are being followed through a large range of air mass, so Eq. 8 can be applied.

Look again at Eq. 8, and the little graph sketched below it. These offer a way to decide if second-order extinction is significantly affecting the night's observations with the selected comp stars. The idea is to use the fact that each star has a slightly different slope on the IM vs. X curve. Call that slope m. According to Eq. 8,

$$m = k'_v + k''_v\cdot(b\text{-}v) \qquad (12)$$

that is, each star gives its own value of m, and if we plot m versus (b-v), we expect to get a linear graph, whose slope is the second-order extinction, and whose y-intercept is the first-order extinction. With modern spreadsheets, that calculation is a trivial exercise.

For example, using the data given in Figure 1, recognizing that each star has slightly different slope, and using the (b-v) color index of each star (measured from the same images that give the IM vs. X curves), we get Figure 5.

That is, this method gives $k'_b = .31$ mag/air mass, and $k''_{bv} = k''_b - k''_v = -.03$ which is virtually identical to the value obtained by the standard method of determining second-order extinction (compare with Table 1).

The reason that this may be a valuable observation, particularly for asteroid photometry, is that MPO Canopus permits the use of up to 5 comp stars. If the target field is imaged in b and v, and if (by luck) the selected comp stars span a wide range of color index, then the plots of IM vs. X contain information for *both* first- and second-order extinction.

Warning: there is a bit of mathematical guile hidden in Eq. 12. In truth, (b-v) is not a constant; it is actually a slowly-varying function of the air mass (X). As a practical matter, however, using the average (b-v) over the air mass range used in a given night's session, and treating it as though it were constant, seems to give reasonable and consistent results.

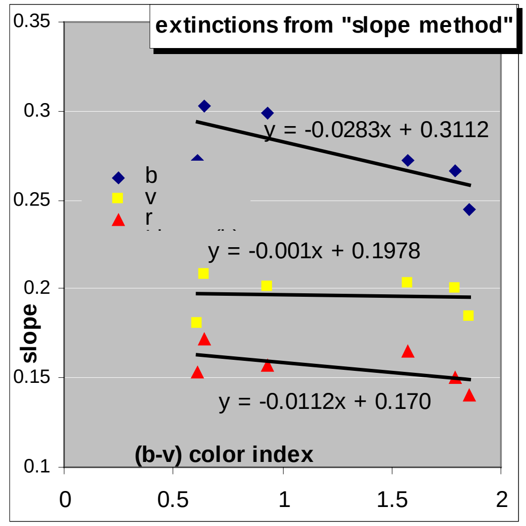


Figure 5: 1st and 2nd order extinctions from "comp star slopes" method

7. Second-order extinctions anchored in (v-r) measurements

The term (b-v) in Eqs. 1, 3, and 5 describes the color of the star (not the band in which it is being observed). While it is standard practice to use the (b-v) color index, there is no fundamental reason that we must describe the star's color by that particular color index. The CCD's that many photometrists use are much more sensitive in v and r bands than they are in b; therefore, it is of interest to examine a variation on the standard definition of second-order extinction that is anchored in the (v-r) color index rather than (b-v). I'll call the first and second-order extinctions so defined as j′ and j″ (to avoid confusion with the conventional extinctions k′ and k″). I was led to this idea by Villata et al (ref 5), who used a second-order extinction definition that is identical to Eq. 15 below.

By analogy to Eq. 1, the defining equations for these alternate second-order extinctions based on (v-r) are:

$$\begin{aligned} v &= v_0 + j'_v\cdot X + j''_v\cdot(v\text{-}r)\cdot X \\ &= v_0 + [j'_v + j''_v\cdot(v\text{-}r)]\cdot X \qquad (13) \end{aligned}$$

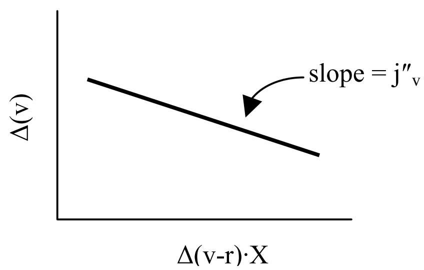
$$\begin{aligned} b &= b_0 + j'_b\cdot X + j''_b\cdot(v\text{-}r)\cdot X \\ &= b_0 + [j'_b + j''_b\cdot(v\text{-}r)]\cdot X \qquad (14) \end{aligned}$$

$$\begin{aligned} r &= r_0 + j'_r\cdot X + j''_r\cdot(v\text{-}r)\cdot X \\ &= r_0 + [j'_r + j''_r\cdot(v\text{-}r)]\cdot X \qquad (15) \end{aligned}$$

The second-order extinction coefficients are measured as before by following a star-pair through a wide range of air masses, applying an equation analogous to Eq. 7:

$$\Delta(v) = j''_v \cdot \Delta(v\text{-}r) \cdot X + \Delta(v)_0 \qquad (16)$$

and constructing the indicated linear plot:



Several questions were investigated for these "alternate" second-order extinctions that are anchored in (v-r) as the defining color index:

- can the j'' be effectively determined? are they consistent?
- are the j'' significantly different from k''? Is the difference understandable?
- are the first-order extinctions $j' \approx k'$ (i.e. is first-order extinction unaffected by the use of j'' vs. k''?)
- are target-object exoatmospheric magnitudes determined by use of j', j'', and (v-r) tolerably identical to exoatmospheric magnitudes determined by the "standard" parameters k', k'', and (b-v)?

The answer to all of these questions turned out to be positive.

The observed range of second-order extinctions, using (v-r) color index to describe the star's color, is shown in Figure 6:

Again, second-order extinctions j''_b and j''_{bv} are negative, and large. This is only of academic interest, since as a practical matter if you're using (v-r) as the reference index, it's probably because you aren't observing through a "b" filter.

The values of j''_v, j''_r, and j''_{vr} may be surprising at first – they seem large enough that it may be wise to account for them in color-index determinations. The reason that they appear to be (relatively) large is that the range of (v-r) is smaller than the range of (b-v) for main-sequence stars. The instrumental color-color diagram (uncorrected for extinction) for the stars

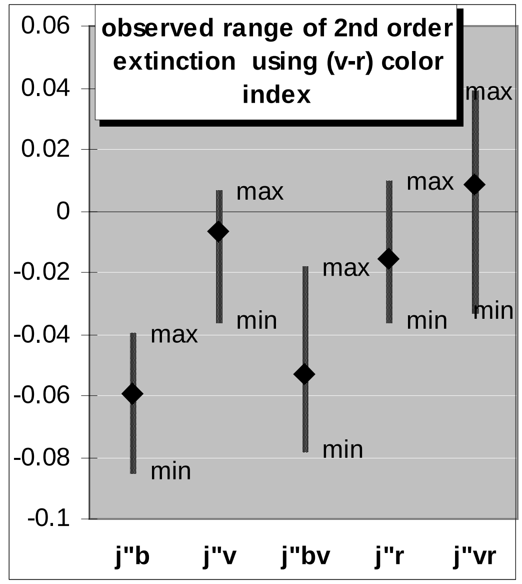


Figure 6: Observed range of 2nd order extinctions using (v-r) as basis color index

used for most of these extinction measurements is very closely matched by:

$$(v\text{-}r) = 0.69\ (b\text{-}v) - .13$$

so that it isn't surprising that the second-order extinction defined using (v-r) is a factor of 1/.69 = 1.4 larger than second-order extinction defined using (b-v).

Considering the difficulty of achieving accuracy better than ±.02 magnitude in most projects, it seems warranted to continue following the conventional wisdom that "second order extinction in v, r, and (v-r) is negligible".

8. Comparison of Altimira Observatory with other sites reporting extinction values

A search for references reporting extinction measurements at professional observatories gave an interestingly wide range of representative extinction values, which are summarized in Table 2:

Table 2: Representative Extinction values reported by professional observatory sites						
Site	**k'_v**	**k'_{bv}**	**k''_v**	**k''_{bv}**	**comments**	**ref**
La Silla			0.00	-.035	“standard” values	3
Bochum	.130±.014	.120±.013	0.00	-.014	1991	3
ESO 50 cm	.119±.040	.134±.010	0.00	-.028		3
CTIO	.140	.090		-.025	affected by El Chincon eruption	4
SPO	.250	.100		-.033		4
UAO	.250	.060		-.033		4
Cerro Tololo	.152	.124		-.023	average (13 year period)	1
Cerro Tololo	.099 - .25	.074 - .184		-.046 to +.013	total range (13 yr period)	1
VBO	.224	.193	0.01	+ .026 (sic)	elev = 725 m ASL	6
Cesco	.160±.055	.141±.024	0±.009	-.032 ± .006	elev = 2500 m ASL	7

ESO = European Southern Observatory
CTIO= Cerro Tololo International Observatory
SPO= Sacramento Peak Cloudcroft site
UAO= University of Arizona Mt. Lemon
VBO= Varnu Bappu Observatory (Kavalu, India)
Cesco= Estacion Astronomica “Dr. Carlos Ulrrico Cesco”, of Felix Aquilar Obs., San Juan, Argentia.

9. Acknowledgements:

Photometric reductions were done using Brian Warner’s MPO Canopus/PhotoRed program. Charting and star identifications were done with Chris Marriott’s SkyMapPro. Automated observatory control was done using the Software Bisque suite: TheSky, Automadome, and CCDSoft.

Sections 2 and 3 of this paper draw liberally from Chapter 4 of Henden and Kaitchuck (ref 2). I used their equations and symbology for the convenience of readers who may want to cross-reference with ref 2. I have also explicitly stated some equations that they left as “exercises for the reader” because the subject of second order extinction is pretty confusing for some of us.

10. References

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